Colloque

Analyse harmonique sur les groupes de Lie et sur les espaces symétriques – en l'honneur de Jacques Faraut

Nancy-Strasbourg, du 10 au 15 juin 2005

Résumés

E.P. van den BAN (Utrecht): A Paley-Wiener theorem for distributions on symmetric spaces. — In this talk I will present a Paley-Wiener theorem for compactly supported distributions on a reductive symmetric space. I will explain the strategy of proof that allows the derivation of this result from an earlier Paley-Wiener theorem for smooth functions The talk is based on joint work with Henrik Schlichtkrull.

L. BARCHINI (Oklahoma): Remarks on the charactersitic cycle of discrete series of $SU(p,q)$. — (Joint work with R. Zierau.) Let G be a connected real semisi55.238H-3.267979205(.)-3.7468(r)0.204213.680Tg054(t)-24 difficult problem in general. In this talk we assume that $G = SU(p,q)$ and that π is a discrete series representation and present some examples an remarks that are related to this general problem.

Let X be the flag manifold of $\mathfrak{g}_{\mathbb{C}}$ and let Z be a closed $K_{\mathbb{C}}$ orbit in X. Then, Z contains a Borel subalgebra $\mathfrak{b} = \mathfrak{h}_{\mathbb{C}} + \mathfrak{n}^-$ with $-\Delta^+(\mathfrak{k}_{\mathbb{C}}, \mathfrak{h}_{\mathbb{C}}) \subset \Delta(\mathfrak{n}^-)$. This Borel subalgebra determines a positive system $\Delta^+(\mathfrak{g}_{\mathbb{C}}, \mathfrak{h}_{\mathbb{C}})$. We agree that $\mathfrak{b} = \mathfrak{h}_{\mathbb{C}} + \mathfrak{n}^-$ has $\mathfrak{n}^-$ is spanned by the negative roots vectors. We denote by ρ half the sums of the positive roots. Let $\lambda \in \mathfrak{h}_{\mathbb{C}}^*$ be regular and $\Delta^+(\mathfrak{g}_{\mathbb{C}}, \mathfrak{h}_{\mathbb{C}})$-dominant and so that $\lambda - \rho$ is the differential of a character ϕ of the corresponding Cartan subgroup $H \subset G$. There is a bijection between the set of triples $(Z, \phi, \Delta^+(\mathfrak{g}_{\mathbb{C}}, \mathfrak{h}_{\mathbb{C}}))$ and the set of representations π_λ in the discrete series with infinitesimal character.

We write the conormal bundle of Z in X as $T_Z^*(X) = K_{\mathbb{C}} \times_{K_{\mathbb{C}} \cap B} (\mathfrak{n}^- \cap \mathfrak{p}_{\mathbb{C}})$ and observe that the moment map γ restricted to $T_Z^*(X)$ is

$$\gamma : K_{\mathbb{C}} \times_{K_{\mathbb{C}} \cap B} (\mathfrak{n}^- \cap \mathfrak{p}_{\mathbb{C}}) \to \mathfrak{g}_{\mathbb{C}}, \quad (k, Y) \mapsto k \cdot Y.$$

The map γ is proper. Thus, $\gamma(T_Z^*(X))$ is a closed irreducible subvariety of $\mathcal{N}_{\mathfrak{k}}^*$. Indeed, $\gamma(T_Z^*(X))$ is the closure of a single $K_{\mathbb{C}}$-orbit, $\mathcal{O}$.

Starting with a triple $((Z, \phi, \Delta^+(\mathfrak{g}_{\mathbb{C}}, \mathfrak{h}_{\mathbb{C}}))$ we give an algorithm to construct a convinient base point in $\mathcal{O}$. We use the algorithm to give a description of the fiber of the moment map in terms of the structure of G.

S. BEN SAID (Aarhus): Things You can do with $Sl(2,\mathbb{R})$. The Segal-Bargmann transform is a standard and important tool in harmonic analysis and mathematical physics. This transform is the intertwining operator between the Schrödinger model of the canonical commutation relations, in which the position and the momentum operators are represented by x and $\frac{i^{-1}\partial}{\partial x}$ and the Fock model, in which the creation and the annihilation operators are represented by z and $\partial/\partial z$. Further, the Segal-Bargmann transform is a unitary map from $L^2(\mathbb{R}^N)$ to the classical Fock space of holomorphic functions on C^N. In this talk we will present a generalization of the Segal-Bargmann transform when the ordinary derivatives are replaced by the Dunkl differential-difference operators. To do so, we introduce a Fock-type space with reproducing kernel equal to the Dunkl kernel. The generalized Segal-Bargmann transform allows to exhibit some relationships between the Dunkl theory in the Schrödinger model and in the Fock model. Moreover, by means of the Dunkl-Laplacian operators, we will give an analogue of the metaplectic representation for the universal covering of $SL(2,\mathbb{R})$, and discuss things that one can do by means of this representation.

E. DAMEK (Wrocław): Asymptotic behavior of Poisson kernels on NA-groups. — (Joint work with Dariusz Buraczewski and Andrzej Hulanicki.) On a Lie group $S = NA$, that is a split extension of a nilpotent Lie group N by a one-parameter groups of automorphisms A, a probability measure μ is considered and treated as a distribution according to which transformations $s \in S$ acting on $N = S/A$ are sampled. Under natural conditions, formulated some thirty years ago, there is a μ-invariant measure m on N. Properties of m have been intensively studied by a number of authors. The talk deals with the situation when $\mu(A) = \mathrm{P}(s_t \in A)$, where $\mathbb{R}^+ \ni t \to s_t \in S$ is the diffusion on S generated by a second order subelliptic, hypoelliptic, left-invariant operator on S. This talk deals with the most general operators of this kind. Precise asymptotic for m at infinity and for the Green function of the operator are given. To achieve this goal a pseudo-differential calculus for operators with coefficients of finite smoothness is formulated and applied.

P. DELORME (Marseille) : H-fixed distribution vectors for induced representations, for a reductive p-adic symmetric space G/H. — (Joint work with P. Blanc.) We establish results for p-adic symmetric space G/H which are analogous to results by E. van den Ban, Olafsson, Brylinski-Delorme and Carmona-Delorme for the real case. Rational functions replace meromorphic functions. Properties of smooth homology of representations on one hand and structural results by G. Helminck -S.P. Wang and A. Helminck-G. Helminck on the other hand are the main tools.

A. DOOLEY (Sydney): Intertwining operators, contractions and the Baum-Connes conjecture. — If $G = KAN$ is the Iwasawa decomposition of a rank-one semi-simple Lie group, it is interesting to use harmonic analysis on N together with the N picture of the principal and exceptional series to analyse the representation theory. In particular, the author recently proved a representation-theoretic version of the Cowling-Haagerup theorem on the approach to the identity by uniformly bounded representations. In order to establish the Baum-Connes conjecture "with coefficients", one needs information about the K picture, and it turns out that this can be obtained from this result together with the study of the contraction of K to NM

M. DUFLO (Paris): On discrete series of Lie groups. — In this lecture, I will recall the classification of discrete series representations of real algebraic Lie groups, in the setting of the orbit method, and discuss related properties of Lie algebras. In the case of reductive groups, I will present some results on their restrictions to a closed subgroup.

M. FUCHSSTEINER (Darmstadt): Lifting Lie Algebra cocycles. — It is shown that for an n-connected infinite dimensional Lie Group G and G-module A with Lie algebras $\mathfrak{g}$ resp. $\mathfrak{a}$ every Lie algebra cocycle $\omega \in Z^n(\mathfrak{g}, \mathfrak{a})$ can be integrated to a locally smooth group cocycle $f_\omega \in Z^n_s(G, A)$ if and only if the $n+1$-th period map $P_{n+1}(\omega) : \pi_{n+1}(G) \to A$ is trivial.

S. GINDIKIN (Rutgers): Harmonic analysis on symmetric spaces from point of view of complex analysis. — About 50 years ago I. Gelfand suggested that the horospherical transform is an universal tool to solve problems of harmonic analysis on homogeneous spaces. Gelfand and Graev gave several spectacular illustrations of this methods but simultaneously several serious restrictions of the method were clear. The most important of them is that discrete series of representations lie in the kernel of the horospherical transform and can not be investigated in such a way.

I want to show that the union of the horospherical method with complex analysis essentially extends the possibilities of the method. Firstly, I want to explain how complex horospheres work for compact groups Lie (where there are now real horospheres) and I believe that it gives an interesting view on this classical subject. I will also talk about several more advanced applications: the separation of series of representations on pseudo Riemannian symmetric spaces, harmonic analysis on nonsymmetric homogeneous spaces, connections with non linear differential equations.

H. GLÖCKNER (Darmstadt) : Ultrametric invariant manifolds and applications in Lie theory. — In 1970, M.C. Irwin initiated a particularly simple and elegant method to prove the stable manifold theorem: the latter can be reduced to the usual implicit function theorem in Banach spaces. Later, Irwin also applied his ideas to the (technically more complicated) case of pseudo-stable manifolds.

In the present talk, I'll recall Irwin's method and explain how it can be adapted to dynamical systems over ultrametric fields.

The ultrametric invariant manifold theorems so obtained admit various applications in Lie theory. In particular, they enable progress concerning Lie groups over local fields of positive characteristic.

References:

[1] Glöckner, H., Invariant manifolds for dynamical systems over ultrametric fields, manuscript in preparation.

[2] Irwin, M.C., On the stable manifold theorem, Bull. London Math. Soc. 2 (1970), 196-198.

[3] Irwin, M.C., A new proof of the pseudostable manifold theorem, J.

London Math. Soc. 21 (1980), 557-566.

P. GRACZYK (Angers): Recent developments around the product formula on symmetric spaces. — It is now well-known that in the product formula for spherical functions

$$\phi_\lambda(e^X)\,\phi_\lambda(e^Y) = \int_a \phi_\lambda(e^H)\,d\mu_{X,Y}(H)$$

on a Riemannian symmetric space of non-compact type, the measure $\mu_{X,Y}$ is absolutely continuous when X, Y are.49205(o)-1.87468(u)tr. k We discuss different
reg.49205(o)-1.8746307(l)-3.27252(a)-1.87468(r)0

we discuss Rang.49205(o)-e Questions for the Fo l transforms on symmetric spaces. The analog.49205(o)-ies between the flat spacel.87468(u)rved spaces pa.87468(r)-ise certa.49205(o)-in g.49205(o)-1.87468(u)esss, con

K.-H. HOFMANN (Darmstadt): A class of infinite dimensional Lie groups whose Lie theory we know. — All con locally compact g.49205(o)-ro , all p.87468(u)roducts of Lie g.49205(o)-roups are p.87468(u)ro-Lie g.49205(o)-ro o presenendn a mon which sho of this proct.

T. KOBAYASHI (Kyoto): Analysis on hyperboloids revisited from theiewpoint of branching laws of unitary representations. — I plan to ta.49205(o)-lk abo unitary representa.49205(o)-tions ofndefi ithmphasis on the relation to harmonic analysis on hypero

A. KORÀNYI (New York): Parabolic g.(e)-.(o).(m).(e)-.(t)-.(r).(y)-.(a)-.(n)-.(d)-.(m)
. Theang.49205(o)-ent b.87468(u)ndle of G-invariant subs.87468(u)-
nducd by the simplerettiveroots associted to P; th
a multicontact structure. Theorem: If P is non-maxmal, ev-
h map of any con G/P that p.87468(u)reserves the
ct structure is the resriction of an etment of G. This is a
lt with. Cowling.49205264(,)-434.885(F)0.341036(.)-414.809(d)3.49205(e)-418.779(M)3.25516(a)-1.87468(r)0.20421
esotherrelated 88(p)-26.6172(e)2364(s)2.98356lts will be discussed.87468(u).

RÖTZ (Kyoto) : onvexity theorems on the complex

crown. — Our concern is with the complex crown, the natural complexification of a Riemannian symmetric space of the non-compact type X. In this talk we will discuss two convexity theorems on the crown related to the action of the isometry group of X. (Work partly joint with S. Gindikin and M. Otto).

V. MOLCHANOV (Tambov) : Canonical and boundary representations on hyperboloids. —

G. OLAFSSON (Baton Rouge) : Jordan Algebra, Representations and Special Functions. — The classical Laguerre functions ℓ_n^λ form an orthogonal basis for the Hilbert space $L^2(\mathbb{R}^+, x^{\lambda-1}dx)$, $\lambda > 0$. The first generalizations of the Laguerre functions and polynomials were in the work of F. Tricomi in 1935 and C. S. Herz in 1955. The generalization to all symmetric cones using Euclidean Jordan algebras was achieved almost 40 years later in the work of J. Faraut and A. Koranyi. Here the Laguerre polynomials were defined in terms of certain polynomials $\psi_{\mathbf{m}}(x)$ invariant under the action of a maximal compact subgroup L leaving the cone invariant and fixing the identity e:

$$L_{\mathbf{m}}^\nu(x) = (\nu)_{\mathbf{m}} \sum_{|\mathbf{n}| \leq \mathbf{m}} \begin{pmatrix} \mathbf{m} \\ \mathbf{n} \end{pmatrix} \frac{1}{(\nu)_{\mathbf{n}}} \psi_{\mathbf{n}}(-x) \, .$$

The Laguerre functions are defined as

$$\ell_{\mathbf{m}}^\nu(x) = e^{-\mathrm{tr}x} L_{\mathbf{m}}^\nu(2x)$$

where tr is the trace in the corresponding Jordan algebra. It was shown, that the Laguerre functions were orthogonal and eigenfunctions of the Hankel transform. Later, F. Ricci and A. Vignati constructed higher order differential operators, in the context of the Jordan algebra of Hermitian symmetric matrices, having the Laguerre functions as eigenfunctions.

In this talk we will discuss recent joint work with M. Aristidou and M. Davidson. We construct explicit second order differential operators which give a differential equation for those functions as well as recursion relations generalizing the classical relations for the Laguerre functions on $\mathbb{R}^+$. Those differential operators are expressed in therm of the Bessel differential operator introduced by J. Faraut and A. Koranyi. The necessary tools are highest weight representations and Jordan algebras.

B. ØRSTED (Aarhus): Berezin transforms for non-holomorphic discrete series. — The Berezin transform is important in connection with quantization on Hermitian symmetric spaces, where it arises in connection with holomorphic discrete series representations. In this talk, we shall describe an extension to other discrete series representations and some applications to branching laws for such representations.

A. PASQUALE (Metz): Θ-hypergeometric and Θ-Bessel functions. — In this talk, we discuss the Θ-spherical functions as solutions of the hypergeometric system of Heckman and Opdam, as well as the Θ-Bessel functions of Ben-Said and Oersted as their suitable limits.

F. RICCI (Pise): Spectral analysis of the Hodge Laplacian on the Heisenberg group. — This is joint work with D. Müller and M. Peloso. Consider a $U(n)$-invariant left-invariant Riemannian metric on the $(2n + 1)$-dimensional Heisenberg group H_n. We prove that, if Δ_1 is the Hodge Laplacian acting on differential 1-forms, and if m is a Mihlin-Hörmander multiplier on the positive half-line, with L^2-order of smoothness greater than $n+\frac{1}{2}$, then $m(\Delta_1)$ is L^p-bounded for $1 < p < \infty$. This follows from an explicit description of the spectral decomposition of Δ_1 on the space of $L2$-forms in terms of the spectral analysis of the sub-Laplacian L and the central derivative T, acting on scalar-valued functions.

H. SCHLICHTKRULL (Copenhague): Paley-Wiener spaces for real reductive Lie groups. — The Paley-Wiener theorem for K-finite compactly supported smooth functions on a real reductive Lie group G is due to J. Arthur (1983). A similar theorem for reductive symmetric spaces has been proved more recently by E. van den Ban and myself. In this talk I will discuss the derivation of Arthur's result by the specialization of our work to G, viewed as a symmetric space. The main difference between the theorems lies in the normalization of the Eisenstein integrals and the associated Fourier transforms.

R. STANTON (Colombus) : Some properties of Faulkner structures. — I shall present some results, joint with M. Slupinski, and some results soon to be, concerning the geometric properties of low-lying nilpotent co-adjoint orbits. Various geometric and invariant theory results related to reconstruction theorems will be presented.

A. STRASBURGER (Warszawa): A new form of the spherical expansion of zonal functions. — (Based partialy on the joint work with A. Bezubik and A. Dabrowska) The talk aims at presenting a new form of an expansion of zonal functions into spherical harmonics together with its derivation form scratch. Unlike the classical approach, which relies on integral identities and the Funk - Hecke theorem, our method uses only the canonical decomposition of homogeneous polynomials and links the spherical expansion of the zonal function directly to the Taylor expansion of its profile function. Some applications of the expansion to problems in Fourier analysis on euclidean space are presented, and possible generalization to the case of compact symmetric spaces of rank one is outlined.

Ch. TOROSSIAN (Paris) : Kontsevich's Quantization and applications to symmetric spaces. — Since Kontsevich's formula for quantization of Poisson varieties, some certain questions around Lie algebras and symmetric spaces have been renewed. The quantization of Kontsevich and its derivative, appear in a number of problems like an alternative to the "methods of orbits" and "the quantization of Kostant-Kirillov-Souriau". We will show in this talk, how to find by these new methods some results around the algebra of differential operators for symmetric spaces. One will also discuss recent progress in this field.

Kh. TRIMÈCHE (Tunis): The transmutation operators in Mathematics. — We consider the transmutation operators introduced in 1956 by J. Delsarte in the University of Nancy. The first examples of such operators are given by J. L. Lions in the same University. In this conference we speak about the links of these operators with the group theory, the hypergroup theory, the Dunkl operators theory, and their usefulness in the construction of harmonic analysis.

Chr. WOCKEL (Darmstadt): Infinite dimensional Lie theory for gauge groups. — Gauge groups occur in mathematical physics as infinite-dimensional symmetry groups of gauge theories. These theories are formulated in terms of a smooth K-principal bundle $q : P \to M$, and the gauge group may be identified with the space of smooth K-invariant mappings $C^\infty(P, K)^K$. If the bundle is trivial or K is abelian, then $C^\infty(P, K)^K$ is isomorphic to $C^\infty(M, K)$, but in general (e.g. for so called Yang-Mills Theories) this is not always the case.

This talk describes how Lie theoretic results for $C^\infty(M, K)$ can be transfered to $C^\infty(P, K)^K$. This will cover central extensions of $C^\infty(P, K)^K$, their actions on $\mathrm{Aut}(P)$ and the calculation of the homotopty groups $\pi_n(C^\infty(P, K)^K)$ for bundles over compact orientable surfaces M.